

Assessment of The Seismic Stress–Strain State of Radial-Beam Domes of The “Eski Hammam” Architectural Monument

A.T. Ilyasov¹, S.K. Mukhammadov²

¹Berdaq Karakalpak State University

²Bukhara State Technical University, Email: s.muhammadov@mail.ru

Abstract. This study investigates the stress–strain state of the sixteenth-century “Eski Hammam” architectural monument under seismic loading, with particular emphasis on its radial-beam domes and the composite membrane proposed for structural strengthening. The main objective of the study was to assess the deformation behaviour of the dome system and the overall building–foundation–soil interaction under static and seismic actions. Numerical modelling was performed using the finite element method implemented in the LIRA-SAPR software package. The bending and longitudinal stiffness of the dome supporting contour in the membrane plane were taken into account, together with empirical relationships describing the deformation behaviour of the strengthening composite membrane.

The numerical results showed that the effective modal mass participation of the restored building model was 90.69% along the X-axis, 91.93% along the Y-axis, and 99.97% along the Z-axis. According to the Richard von Mises model, the maximum displacement of the restored structure did not exceed 158.25 mm, while the calculated foundation settlement was 6.1 cm. These values were compared with the adopted regulatory limits and indicated satisfactory seismic performance of the restored structural system. The study also developed practical recommendations for strengthening historical domes with composite membranes, including membrane attachment to the supporting contour, corrosion protection, prevention of local crushing at connection zones, and provision of effective drainage of precipitation from the deformed membrane surface.

Keywords: architectural monument, Eski Hammam, radial-beam dome, seismic loading, finite element method, LIRA-SAPR, composite membrane, stress–strain state, foundation settlement, soil–structure interaction, restoration.

Introduction

The preservation and structural safety assessment of historical architectural monuments represent important problems in modern structural engineering and conservation practice. Unlike contemporary buildings, historical structures often have complex geometries, masonry load-bearing systems, vaulted and dome-shaped spatial structures, and material properties that have changed as a result of long-term operation, environmental exposure, previous repairs, and restoration interventions.

Architectural background for this assessment includes Atoshov’s dissertation on the architecture and typological solutions of Ichan Qala objects [1]. Djumakulov’s dissertation addresses geometric harmonization in traditional Central Asian architecture and urban planning [2]. Seismic risk across different structural systems is the subject of Kabirov’s master’s thesis [3]. These sources provide the architectural and seismic context for examining a historical dome structure.

Earlier dome research includes Durakov’s experimental and numerical investigation of spherical reticulated domes [4]. Savelev examined the strength and stability of large-span metal reticulated domes [5]. Tolushov considered analysis methods and structural solutions for reticulated timber domes [6]. Fadeev investigated the stability of reticulated spherical shell roofs through

model tests under short-term loading [7]. These studies concern different materials and structural configurations, so their direct applicability to the Eski Hammam system requires consideration of its geometry and construction.

Loading assumptions are also relevant to the choice of a dome model. Shapin and Ovsiyannikov addressed spherical framed domes under axisymmetric loading [8]. Shapin subsequently considered framed domes under arbitrary loading [9]. Savelev's work also addressed the stability of reticulated domes [10]. Skuratov examined the theoretical foundations for designing timber polygonal domes [11]. The general mechanics background includes Timoshenko's treatment of strength of materials [12]. Plate and shell behaviour is covered by Timoshenko and Woinowsky-Krieger [13]. Together, these subjects frame the need to distinguish general structural theory from the specific assumptions and calculated response of the monument examined here.

The seismic response of historical dome structures is particularly dependent on their geometric configuration, the stiffness of the supporting contour, soil–structure interaction, and the spatial behaviour of the individual dome components. Therefore, assessment based solely on simplified structural schemes may not adequately represent the actual behaviour of such buildings. Advanced numerical modelling that considers the interaction between the building, foundation, and soil provides a more reliable approach for determining the actual stress–strain state of historical structures.

The object of the present study is the “Eski Hammam” architectural monument, constructed in the sixteenth century. The study focuses on the seismic behaviour of its radial-beam domes and the deformation response of a strengthening composite membrane. The structural response of the building was evaluated under static and dynamic loading, taking into account the foundation and soil conditions.

The main objective of the research is to determine the seismic stress–strain state of the historical building, evaluate the deformation behaviour of its radial-beam domes, assess the effectiveness of a composite strengthening membrane, and develop engineering recommendations for the restoration and strengthening of historical dome structures.

The scientific novelty of the study lies in the integrated assessment of the seismic response of a historical dome structure by considering the stiffness of its supporting contour, the deformation characteristics of the strengthening composite membrane, and the interaction between the building, foundation, and soil within a unified numerical model.

Materials and Methods

Numerical model

General computational techniques for mathematical calculations are described by Forsythe, Malcolm, and Moler [14]. An account of finite element applications in structural analysis is provided by Hechumov, Keppler, and Prokopyev [15].

The structural model of the “Eski Hammam” building was developed for static and seismic loading conditions using the finite element method. The model included the principal structural components of the building, radial-beam domes, supporting contours, foundations, and soil interaction.

The numerical analyses were performed using the LIRA-SAPR software package. Separate computational models were developed for static loads, seismic loads, and different combinations of seismic actions.

Six-component accelerograms were applied along the X, Y, and Z coordinate directions. The deformation coefficient of the masonry was adopted as:

$$\xi = 0,025$$

The effective modal mass participation was also monitored during the analysis. The results demonstrated that the effective mass participation in the horizontal seismic directions was not less than 90% of the total structural mass, while the corresponding value in the vertical direction was not less than 75%.

Consideration of the stiffness of the dome supporting contour

The seismic deformation of the radial-beam domes was evaluated by taking into account the bending and longitudinal stiffness of the dome supporting contour in the membrane plane.

The bending and longitudinal stiffnesses of the supporting contour were determined from:

$$E_{1k}=E_{1a}/N_0, E_{ak}=E_{1a3}/N_0$$

Here, E_{1a} represents the initial stiffness of the supporting contour, (N_0) is a structural parameter, while E_{1k} and E_{ak} represent the bending and longitudinal stiffnesses, respectively. For the upper belt of the dome, the inclination angle was adopted as:

$$\rho = 15^\circ$$

with:

$$\delta_1 = \delta_2$$

These geometric parameters were considered appropriate for the investigated radial-beam dome system.

Determination of dome deformation

The maximum displacement and stresses in the dome elements were determined using empirical relationships derived from finite element analyses. The initial deformation of the strengthening composite membrane was not considered.

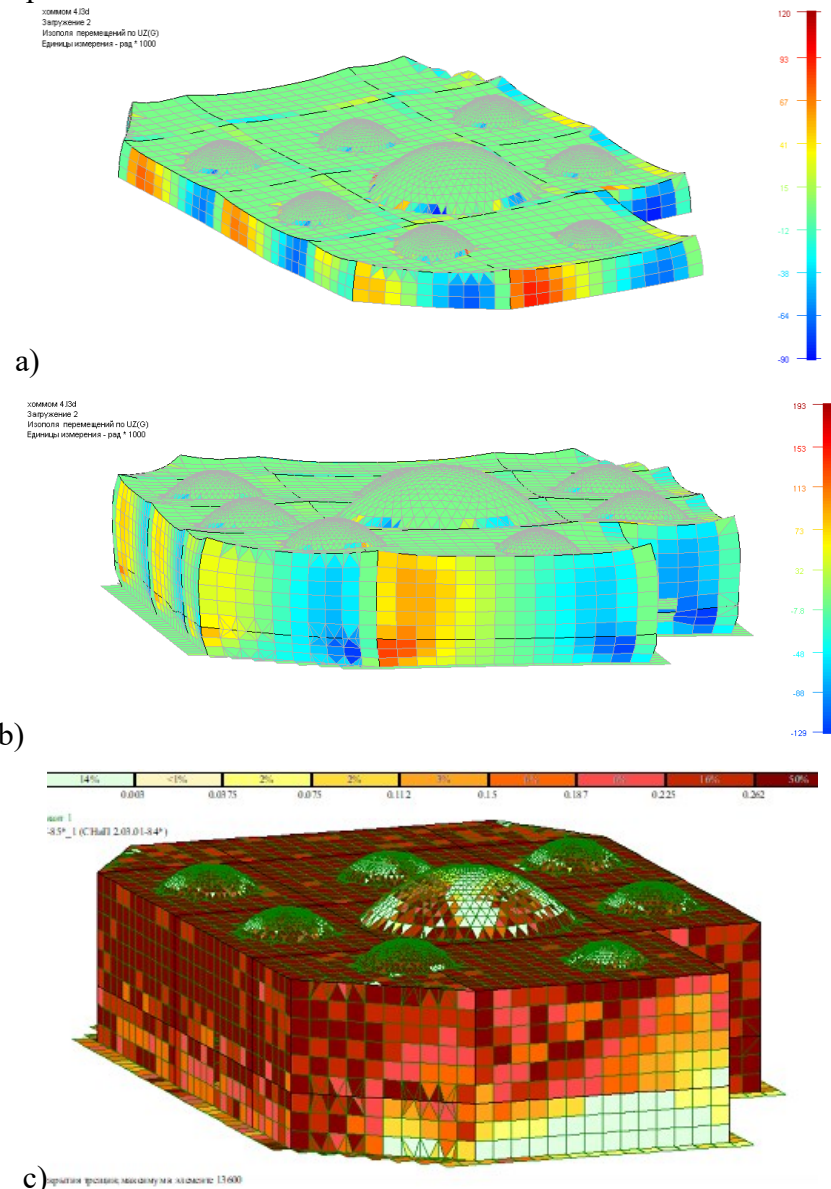


Figure 1. Computational models of the “Old Bathhouse” building, constructed in the 16th century: (a) computational model under static loads; (b) computational model under seismic loads; (c) computational model accounting for various combinations of seismic effects.

The maximum displacement was calculated as:

$$w_{\max} = \alpha_1 * (m^2 a^2 / E)$$

$$\sigma_{\max} = \gamma_1 * (m^2 a^2 / E)$$

$$\tau_{\max} = \gamma_3 * (m^2 a^2 / E)$$

while the maximum normal and shear stresses were determined from:

The coefficient α_1 was determined as follows.

$$0.00001 < n < 0.001: \alpha_1 = 10360 n^2 - 23.334 n + 0.0809$$

$$0.001 < n < 0.1: \alpha_1 = 0.0214 \cdot n^{(-0.171)} \cdot (k + 1) \cdot e^{(0.0273k)}$$

For γ_1 coefficient

$$0.00001 < n < 0.001, k < 1: \gamma_1 = 0.0835 \cdot n^{(-0.089)} \cdot (k + 1)$$

$$0.001 < n < 0.1, k < 1: \gamma_1 = 0.044 \cdot n^{(-0.185)} \cdot (k + 1) \cdot e^{(0.0173k)}$$

$$0.00001 < n < 0.001, 1 < k < 20: \gamma_1 = 0.0835 \cdot n^{(-0.089)} \cdot (k + 1)$$

$$0.001 < n < 0.1, 1 < k < 20: \gamma_1 = 0.044 \cdot n^{(-0.185)} \cdot (k + 1)$$

The shear stress coefficient γ_3 was determined as follows

$$0.00001 < n < 0.001: \gamma_3 = 0.0607 \cdot n^{(-0.09)} \cdot (k + 1) \cdot e^{(0.0154k)}$$

$$0.001 < n < 0.1: \gamma_3 = (-0.017 \ln n - 0.0052) \cdot (k + 1)$$

Building–foundation–soil interaction

To approximate the actual operating conditions of the historical building, the physical and mechanical properties of the foundation soil were incorporated into the numerical model.

The Pasternak and Mohr–Coulomb soil models were employed to evaluate the influence of soil deformation characteristics on the structural response.

The following parameters were determined:

- Displacements along the X, Y, and Z axes;
- Foundation settlement;
- Bending moments caused by seismic loading;
- Longitudinal forces;
- Contact pressure transmitted from the building to the soil.

Three structural conditions were analysed: the pre-damage condition, the damaged condition, and the restored condition.

Result

The numerical analysis enabled a comparative assessment of the structural behaviour of the “Eski Hammam” building in its pre-damage, damaged, and restored conditions.

For the restored building model, the effective modal mass participation was:

90.69% along the X-axis;

91.93% along the Y-axis;

99.97% along the Z-axis.

These values satisfy the adopted requirements for seismic analysis.

Table 1. Comparison of displacements and internal forces in the “Eski Hammam” building under static and seismic loading

Calculation model	Structural condition	Maximum displacement, mm	Soil resistance, t/m ²	Building load on soil, t/m ²	Foundation settlement, cm
Pasternak model, static load	Pre-damage	–	16.06	5.18	3.94
Richard von Mises model, dynamic load	Pre-damage	158.25	16.06	15.4	6.62
Richard von Mises model, dynamic load	Damaged	105	16.06	9.85	3.88
Richard von Mises model, dynamic load	Restored	158.25	16.06	17.4	6.10
Mohr–Coulomb model, dynamic load	Pre-damage	158.25	16.06	16.0	8.59
Mohr–Coulomb model, dynamic load	Damaged	105	16.06	14.9	4.11
Mohr–Coulomb model, dynamic load	Restored	158.25	16.06	17.4	6.60

According to the Pasternak model, the calculated seismic displacements varied from **95.7 to 241 mm** along the X-axis and from **41.8 to 291 mm** along the Y-axis.

According to the Richard von Mises model, the corresponding displacements were lower, ranging from **92 to 150 mm** along the X-axis and from **78.7 to 186 mm** along the Y-axis.

The analysis of soil loading showed that, according to the Pasternak model, the pressure transmitted to 1 m² of soil varied from **9.85 to 17.4 t/m²**. According to the Mohr–Coulomb model, the corresponding values ranged from **14.9 to 17.4 t/m²**.

According to the adopted requirements of QMQ 2.01.03-19, the permissible displacement along the X and Y axes was determined as:

$$\Delta_{x,y} = H/200 = 31650/200 = 158,25 \text{ mm}$$

For the restored building model, the maximum displacement according to the Richard von Mises model did not exceed **158.25 mm**.

The calculated foundation settlement of the restored structure was **6.1 cm** according to the Richard von Mises model and **6.6 cm** according to the Mohr–Coulomb model. These values fall within the adopted permissible settlement range of 6–10 cm.

The soil pressure under the restored building reached **17.4 t/m²**, which corresponds to the adopted experimental soil resistance value. Therefore, the interaction between the restored structural system and the foundation soil should be subject to continued monitoring.

Discussion

The obtained results demonstrate that the seismic response of the “Eski Hammam” building should be evaluated not only on the basis of the stiffness of its superstructure but also by considering

the deformation characteristics of the foundation and soil.

The higher displacements obtained in some cases using the Pasternak model compared with the Richard von Mises model indicate that the assumed deformation characteristics of the soil have a significant influence on the overall seismic response of the structure. Therefore, the selection of an appropriate soil model is an important factor in the seismic assessment of historical buildings.

The results obtained using the Richard von Mises criterion indicate that the restored building operates within the adopted permissible displacement limit. The effective modal mass participation values of 90.69% in the X direction, 91.93% in the Y direction, and 99.97% in the Z direction demonstrate that the numerical model adequately represents the principal dynamic characteristics of the structure.

At the same time, strengthening interventions in historical structures should not be aimed solely at increasing structural strength. Preservation of the architectural and historical character of the monument must also be considered. From this perspective, the use of a composite strengthening membrane attached to the supporting contour of the radial-beam dome provides an opportunity to increase structural stiffness while minimising intervention into the original historical fabric.

The strengthening composite membrane can be attached to the supporting contour using welded angle connections or self-tapping screws. When screw connections are used, a relatively small installation spacing is recommended. This approach reduces local crushing in the connection zone and decreases the overall flexibility of the membrane sheet.

To prevent wrinkling and excessive internal stresses in the membrane, installation should begin at the central portion of the supporting contour and proceed gradually toward the edges. Such a sequence provides more uniform membrane tension and reduces the risk of local deformation.

Corrosion protection is another important factor affecting the long-term reliability of the strengthening system. Appropriate protective measures should be selected according to the aggressiveness of the external and internal environment. Particular attention should be paid to crevice corrosion in overlapping connections between the membrane and the supporting contour. Such gaps should be reliably sealed using suitable sealing materials.

In addition, accumulation of precipitation water on the deformed membrane surface should be prevented. For this purpose, the slope angle of the tangent plane to the deformed membrane surface should remain positive. If the tangent slope becomes negative, local water-accumulation zones may develop. In such cases, the following measures should be considered:

- a. Increasing the overall stiffness of the structural system;
- b. Recalculating the membrane geometry;
- c. Reconsidering the distribution of applied loads.

Conclusion

The seismic stress–strain state of the “Eski Hammam” architectural monument was investigated using comprehensive numerical modelling.

The following conclusions can be drawn from the study:

The assessment of the seismic safety of the “Eski Hammam” monument requires comprehensive modelling of the building–foundation–soil system to determine the actual stress–strain state of the structure.

The finite element method implemented in the LIRA-SAPR software package provides an effective approach for analysing complex spatial structural systems of historical buildings, including radial-beam domes, under seismic loading.

For the restored “Eski Hammam” model, the effective modal mass participation was 90.69% along the X-axis, 91.93% along the Y-axis, and 99.97% along the Z-axis, satisfying the adopted requirements for seismic analysis.

According to the Richard von Mises model, the maximum displacement of the restored structure did not exceed 158.25 mm. This value corresponds to the permissible displacement determined by the relationship:

$$\Delta_{x,y} = H/200 = 31650/200 = 158,25 \text{ mm}$$

The calculated foundation settlement of the restored building was 6.1 cm according to the

Richard von Mises model, which falls within the adopted permissible settlement range.

The differences between the Pasternak and Mohr–Coulomb model results demonstrate that an appropriate representation of soil deformation characteristics is important for the seismic assessment of historical structures.

Strengthening radial-beam domes by attaching a composite membrane to the supporting contour can be considered an effective method for increasing structural stiffness and limiting deformation.

Installing the composite membrane progressively from the central part of the supporting contour toward the edges can reduce local crushing at connections and prevent excessive wrinkling of the membrane.

Protection of membrane-to-contour connections against corrosion, reliable sealing of gaps, and effective drainage of precipitation are essential for ensuring the long-term durability of the strengthening system.

Before restoration of historical architectural monuments, it is recommended to analyse previous engineering investigations, perform engineering-geological and seismic studies, determine the condition of construction materials and structural elements, and apply advanced numerical modelling methods rather than relying exclusively on simplified normative calculations.

Overall, the study demonstrates that the methodology applied to the “Eski Hammam” monument can be used for the technical assessment, seismic evaluation, restoration, and strengthening of other historical dome-shaped architectural monuments. The proposed approach makes it possible to combine structural safety requirements with the preservation of the original architectural and historical character of cultural heritage structures.

References

- [1] S. B. Atoshov, “Architecture of Ichan Qala objects and specific features of their typological solutions,” dissertation, n.d.
- [2] F. U. Djumakulov, “Study and improvement of geometric harmonization principles of traditional architecture and urban planning of Central Asia,” dissertation, n.d.
- [3] I. R. Kabirov, “Investigation of seismic risks for buildings with different structural systems,” master’s thesis, Tashkent Institute of Architecture and Civil Engineering, Tashkent, Uzbekistan, 2011.
- [4] B. N. Durakov, “Experimental and numerical investigation of spherical reticulated domes,” abstract of candidate of technical sciences dissertation, Leningrad, USSR, 1971.
- [5] V. A. Savelev, “Strength and stability of large-span metal reticulated domes,” abstract of candidate of technical sciences dissertation, Moscow, USSR, 1966.
- [6] S. A. Tolushov, “Methods of analysis and improvement of structural solutions for reticulated timber domes,” abstract of candidate of technical sciences dissertation, Penza, Russia, 2002.
- [7] A. A. Fadeev, “Experimental investigation of the stability of reticulated spherical shell roofs using models under short-term loading,” in Proc. All-Union Conf. Thin-Walled and Spatial Structures of Building Roofs, Tallinn, USSR, 1986, vol. 2, pp. 69–70.
- [8] V. M. Shapin and G. E. Ovsyannikov, Analysis of Spherical Framed Domes under Axisymmetric Loading. Minsk, USSR, 1978.
- [9] V. M. Shapin, “Analysis of framed domes under arbitrary loading,” Construction Engineering, Technology, Organization and Economics, no. 6, pp. 121–127, 1980.
- [10] V. A. Savelev, Stability of Reticulated Domes. Moscow, USSR: Stroyizdat, 1966.
- [11] S. V. Skuratov, “Theoretical foundations for the design of timber structures of incomplete polygonal domes,” abstract of candidate of technical sciences dissertation, Rostov-on-Don, USSR, 1990.
- [12] S. P. Timoshenko, Strength of Materials. Moscow, USSR: State Publishing House of Technical and Theoretical Literature, 1955.
- [13] S. P. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, 2nd ed. Moscow, USSR: Nauka, 1966.
- [14] G. Forsythe, M. Malcolm, and C. Moler, Computer Methods for Mathematical Computations.

Moscow, USSR, 1980.

[15] R. A. Hechumov, H. Keppler, and V. I. Prokopyev, Application of the Finite Element Method in Structural Analysis. Moscow, Russia: Association of Higher Educational Institutions of Civil Engineering, 1994.