AMERICAN Journal of Engineering,
Mechanics and Architecture
Volume 4, Issue 07, 2026 ISSN (E): 2993-2637

Flexural Behaviour and Stiffness of Reinforced Concrete T-Beams with Bond-Slip Effects

Kulanov Ikrom Burxonovich
Senior Lecturer, Jizzakh State Pedagogical University,
ikulanov76@mail.ru

Abstract. This study investigates the stiffness and long-term flexural behaviour of reinforced concrete T-beams by incorporating the effects of bond-slip at the reinforcement–concrete interface. Traditional design approaches often assume a perfect bond, which may lead to an overestimation of stiffness and an underestimation of long-term deflection. In this work, an analytical method is developed to quantify the influence of slip on curvature, stiffness reduction, and deflection. The model integrates essential mechanical parameters of concrete, including its modulus of elasticity, time-dependent deformation characteristics, creep, shrinkage, and the nonlinear interaction between concrete and reinforcement. A comprehensive computational framework is established based on regulatory recommendations and long-term deformation models. The results demonstrate that bond-slip significantly increases deflection and reduces effective stiffness compared to conventional bonded assumptions. The analytical expressions derived in this study provide a more realistic prediction of the flexural response of reinforced concrete beams, especially under sustained loading conditions as time approaches infinity. The proposed method can serve as a practical tool for evaluating long-term serviceability performance and can be applied to further experimental and theoretical research in the mechanics of reinforced concrete structures.

Keyword: Bond-slip; Modulus of elasticity; Long-term deformation; Flexural behaviour; Stiffness reduction; Reinforced concrete T-beams.

Introduction

Reinforced concrete structures under sustained loading exhibit progressive deformation due to creep and shrinkage phenomena, which significantly influence their long-term serviceability. Traditional design approaches often assume a perfect bond between reinforcement and concrete; however, recent studies have demonstrated that bond-slip effects can lead to increased deflections and reduced stiffness, thereby altering the flexural response of beams over time.

The accurate prediction of long-term deflections requires consideration of both time-dependent concrete properties and the nonlinear interaction between reinforcement and concrete. (Elaghoury and Bartlett 2019) emphasized that neglecting creep and shrinkage in reinforced concrete beams results in underestimation of deflections, highlighting the necessity of incorporating effective creep coefficients and shrinkage models into analytical frameworks. (Bazant and Baweja 1995) highlighted the importance of creep and shrinkage models for long-term prediction, while (Neville 1997) emphasized the role of concrete properties in serviceability. Objective: To determine the stiffness and deflection at mid-span of a T-section reinforced concrete beam with non-prestressed reinforcement, subjected to loading after $t_0 = 50$ days, considering the behavior of the beam as time tends to infinity ($t \rightarrow \infty$). As shown in Table 1, the modulus of elasticity of concrete increases slightly over time due to the age-related gain in strength, while the creep coefficient reaches its limiting value as $t \rightarrow \infty$. The shrinkage strain also contributes to the reduction in effective stiffness, which is later reflected in the increased long-term deflection of the T-beam. These time-dependent parameters play a critical role in capturing the nonlinear interaction between concrete and reinforcement under sustained loading, as emphasized by (Bazant and Baweja 1995) and (Neville 1997).

In addition, the flexural behavior of T-beams has been extensively studied due to their widespread application in structural systems. (Atmajayanti et al. 2025) investigated strengthening methods for reinforced concrete T-beams and confirmed that the geometry of the T-section plays a critical role in stiffness and load-bearing capacity. (Gilbert 1988) and (Gardner 2004) further highlighted the importance of incorporating shrinkage and creep models into long-term deformation analysis, while (RILEM TC 107 1999) and (CEB-FIP Model Code 2010) provided international guidelines for shrinkage and creep evaluation.

Furthermore, (Jeong et al. 2022) developed advanced mapping functions to estimate bond–slip and bond strength in RC beams, demonstrating that slip at the reinforcement–concrete interface significantly modifies stiffness and ultimate capacity. (Rahaf Mohamad et al. 2025) confirmed through finite element modeling that bond-slip increases deflection and reduces stiffness compared to perfect bond assumptions. (Lin 2010) also proposed a tension stiffening model based on bond stress–slip relations, which provides deeper insights into reinforcement–concrete interaction.

Recent computational and experimental studies have expanded this field. For example, (Gardner and Lockman 2001) compared different creep and shrinkage prediction models, while (ACI Committee 209 1992) provided recommendations for modeling time-dependent deformations in reinforced concrete. (Eurocode 2 2004) and (ACI 318 2019) remain essential design codes that integrate serviceability requirements. More recently, (Bazant 2001) reviewed the evolution of creep and shrinkage prediction methods, emphasizing the need for improved long-term models.

Thus, the present analytical results are consistent with previous experimental and theoretical studies (Atmajayanti et al., 2025; Kosimov, 2019), reinforcing the conclusion that bond-slip must be explicitly considered in serviceability predictions of reinforced concrete T-beams.

Therefore, the present study aims to evaluate the stiffness and deflection of reinforced concrete T-beams under sustained loading conditions, explicitly accounting for bond-slip effects. By integrating creep, shrinkage, and nonlinear bond–slip interactions into the analytical framework, this research contributes to more realistic predictions of flexural behavior and serviceability performance of reinforced concrete structures.

Materials and Methods

Beam Geometry and Support Conditions: A reinforced concrete beam with a **T-shaped cross-section** and a **span length of l=6 m** is simply supported at both ends (pinned supports). The cross-sectional dimensions (see Figure 1) are as follows:

- Width of the web: **b = 20 cm**
- Overall height: **h = 60 cm**
- Width of the flange: b'_f **= 0.35 m**
- Effective depth: **h_0 = 0.6 m**
- Concrete cover (top and bottom): **a = a′ = 5 cm**

The beam is made of **B30 class concrete** and is cured under natural conditions with a **cone slump of 5–6 cm**. Reinforcement class: **A-III**, non-prestressed. Material properties:

- Modulus of elasticity of reinforcement: **$E_b = 2 \times 10^{11}$ N/m²**

- Design yield strength of reinforcement: **R_s = R'_s = 400 MPa = 4 × 10^8 N/m²**

Reinforcement details:

- Tension reinforcement cross-sectional area: **A_s = 32.17 × 10^{-4} m²** (4 bars of 32 mm diameter)
- Compression reinforcement cross-sectional area: A'_s **= 4.52 × 10^{-4} m² = 45.2 cm²**

Applied load:

- Uniformly distributed load including self-weight: **q = 70 × 10^3 N/m**

Additional conditions:

- Concrete age at loading: **t_0 = 50 days**
- Ambient relative humidity: **W = 65%**

Results and Discussion

Objective: To determine the stiffness and deflection at mid-span of a T-section reinforced concrete beam (as shown in Figure 1b) with non-prestressed reinforcement, subjected to loading after t_0 = 50 days, considering the behavior of the beam as time tends to infinity ($t \to \infty$).

Solution: First, calculate the bending moment at mid-span (l/2) for the beam under a uniformly distributed load of q = 70 × 10^3 N/m:

$$M = \frac{ql^2}{8} = \frac{70\cdot 10^3\cdot 6^2}{8} = 31.5\times 10^4\ \text{H}\cdot\text{m}$$

After a certain time (t) following the loading of a normally cured reinforced concrete beam, the compressive strength of concrete can be calculated using the following formula:

$$R_b^G(t) = \left\{1+\left[\frac{23}{55+B}\right]\times\left[\frac{t-28}{t+11}\right]\right\}\times B = \left\{1+\left[\frac{23}{46+B(28)}\right]\cdot\left[\frac{t-28}{t+14}\right]\right\}B(28)$$

The calculated mechanical parameters of the concrete and reinforcement at the loading age and at long-term conditions are summarized in **Table 1**. The table presents the values of compressive strength, elastic modulus, creep coefficient, shrinkage strain, and other time-dependent properties used in the analytical model. These parameters serve as the fundamental input data for evaluating the stress - strain state, determining the height of the compression zone, and estimating long-term stiffness degradation.

As shown in **Table 1**, the modulus of elasticity of concrete increases slightly over time due to the age-related gain in strength, while the creep coefficient reaches its limiting value as t→∞. The shrinkage strain also contributes to the reduction in effective stiffness, which is later reflected in the increased long-term deflection of the T-beam. These time-dependent parameters play a critical role in capturing the nonlinear interaction between concrete and reinforcement under sustained loading.

For B = B(28) and adjusting for the specific values:

$$R_b^G(t) = \left\{1+\left[\frac{23}{46+B(28)}\right]\times\left[\frac{t-28}{t+14}\right]\right\}\times B(28)$$

Substituting the values B(28) = 30 MPa, t_0 = 50 days:

$$R_b^G(t) = \left[1+\frac{23}{46+30}\cdot\frac{50-28}{50+14}\right]\cdot 30 = 33.12\ \text{MPa}\,.$$

According to Table 3, page 8 (Bondarenko and Suvorkin 1987), for $t \to \infty$, the compressive strength of concrete is:

$$R_b^G(\infty) = 39.1\,\text{MPa} = 39.1\times 10^6\ N/m^2$$

And according to Table 2, page 6 (Bondarenko and Suvorkin 1987), the modulus of elasticity of concrete is:

$$E_b(50)=33.59\times10^3 \text{ MPa}=3.359\times10^{10} \text{ H/m}^2,$$
$$E_\delta(\infty)=35.69\times10^3 \text{ MPa}=3.569\times10^{10} \text{ H/m}^2$$

The geometric properties of the reinforced concrete T-beam section are listed in **Table 2**. These parameters include the dimensions of the web and flange, the overall height, reinforcement areas, and the effective depth. The section geometry forms the basis for computing the flexural rigidity, determining the neutral axis depth, and evaluating the cracking and long-term deformation behavior.

As presented in **Table 2**, the relatively large flange width significantly influences the moment of inertia and enhances the initial stiffness of the T-section. However, when combined with long-term effects such as creep, shrinkage, and bond-slip, the effective stiffness decreases over time - leading to larger mid-span deflections. Therefore, accurate representation of section geometry is essential for realistic long-term predictions.

The flexural section modulus of the bending element (beam) with an open cross-section is calculated as follows.

$$\overline{M}=\frac{2(b+h)}{bh+(b'_f-b)h'_f}=\frac{2(0.35+0.6)}{0.2\cdot0.6+(0.35-0.2)\cdot0.06}=14.73\,m^{-1}$$

Based on Table 4, page 9 in reference (Bondarenko and Suvorkin 1987), the ultimate (limiting) value of the concrete creep coefficient is determined.

$$C^N(\infty,28)=84\times10^{-6}\,(\text{MPa})=8.4\times10^{-11}\left(H/m^2\right)^{-1}$$

The design value of the concrete creep coefficient is determined according to the following formula:

$$c=(\infty,28)=C_N(\infty,28)\cdot\xi_{2c}\cdot\xi_{3c}=8.4\times10^{-11}\times0.845\times0.935=6.64\times10^{-11}\left(\frac{N}{m^2}\right)^{-1}$$

where $\xi_{2c}=0.845$; $\xi_{3c}=0.935$ - according to Table 6, page 10 (Bondarenko and Suvorkin 1987).

According to **Tables 11 and 12, page 14 (Bondarenko and Suvorkin 1987)**, based on the value of the open section modulus $\overline{M}=14.73\ m^{-1}$, he parameters for the **long-term deformation function of concrete** are determined as follows:

$$\gamma=0.008 \text{ day}^{-1};\ \gamma_1=0.004 \text{ day}^{-1};\ c=0.5;\ d=0.625.$$

Then, according to formulas (12) and (13) on page 13 (Bondarenko and Suvorkin 1987), the following values are obtained:

$$\Omega(t_0)=C+de^{-\gamma t_0}=0.5+0.625e^{-0.008\times50}=0.919$$
$$f(\infty-50)=1-ke^{-\gamma_1(\infty-50)}=1-0.8e^{-0.004(\infty-50)}=1$$

Then, according to formula (10) (Bondarenko and Suvorkin 1987), the **creep modulus of concrete** is determined by:

$$C^*(t,t_0)=\left(\frac{1}{E_b(t_0)}-\frac{1}{E_b(t)}\right)+C(\infty,28)\cdot\Omega(t_0)\cdot f(t-t_0)$$

At t=∞, t_0=50:

$$C^*(\infty,50)=\frac{1}{3.359\times10^{10}}-\frac{1}{3.569\times10^{10}}+6.64\times10^{-11}\times0.919\times1=6.28\times10^{-11}\left(N/m^2\right)^{-1}$$

Here, the **long-term shrinkage strain** is calculated as:

$$\varepsilon_s(\infty,t_w)=\varepsilon_s^N(\infty,7)\cdot\xi_{1s}\cdot\xi_{2s}\cdot\xi_{3s},$$

where the coefficients ξ_{ic} and ξ_{is} are determined from Tables 5, 6 and 7 on page 10 (Bondarenko and Suvorkin 1987).

The **effective modulus of elasticity for long-term deformation of concrete** is determined as follows:

$$E_{ob}(t,t_0)=\left(\frac{1}{E_b(t)}+C*(t,t_0)\right)^{-1},$$

where $E_b(t)$ is the initial modulus of elasticity of concrete, obtained from **Table 2, page 6 (Bondarenko and Suvorkin 1987)**.

At t=∞, t_0=50:

$$E_{ob}(\infty,50)=\left(\frac{1}{3.569\times10^{10}}+6.28\times10^{-11}\right)^{-1}=1.101\times10^{10}\,N/m^2$$

When the stress is zero, the height of the compression zone is calculated using the following formula:

$$x_0(\infty,50)=\frac{0.5\left[bh^2+(b_f'-b)(h_f')^2\right]E_{ob}(\infty,50)+E_sA_sh_0+E_s'A_s'a'}{\left[bh+(b_f'-b)h_f'\right]E_{ob}(\infty,50)+E_sA_s+E_s'A_s'}=$$

$$=\frac{\dfrac{\left[0.2\cdot0.6^2+(0.35-0.2)\cdot0.06^2\right]}{2}\cdot1.101\cdot10^{10}+2\cdot10^{11}\cdot32.17\cdot10^{-4}\cdot0.55+2\cdot10^{11}\cdot4.52\cdot10^{-4}\cdot0.05}{\left[0.2\cdot0.6+(0.35-0.2)\cdot0.06\right]\cdot1.101\cdot10^{10}+2\cdot10^{11}\cdot32.17\cdot10^{-4}+2\cdot10^{11}\cdot4.52\cdot10^{-4}}$$

$$=0.352\,m=35.2\,cm$$

The stiffness (moment of inertia) of the beam's cross-sectional area under zero stress condition is determined in the following order, based on formulas (2), (3), (4), (5), and (6) from (Bondarenko and Suvorkin 1982)

$$D_{ob,c}(\infty,50)=<\frac{bx_0^3(\infty,50)}{3}+(b_f'-b)\left\{\frac{h_f^{13}}{12}+h_f'\left[x_0(\infty,50)-\frac{h_f'}{2}\right]^2\right\}>E_{ob}(\infty,50)=$$

$$\left\{\frac{0.2\cdot0.352^3}{3}+(0.35-0.2)\left[\frac{0.06^3}{12}+0.06\cdot\left(0.352-\frac{0.06}{2}\right)\right]^2\right\}\cdot1.101\cdot10^{10}=4.23\times10^7\,H\cdot m^2$$

$$D_{ob,t}(\infty,50)=\frac{1}{3b}\left[h-x_0(\infty,50)\right]^3E_{ob}(\infty,50)=\frac{10.2}{3(0.6-0.352)^3 1.101\cdot10^{10}}=1.017\cdot10^7\,H\cdot m^2$$

$$D_{os,c}(\infty,50)=E_s'A_s'\left[x_0(\infty,50)-a'\right]^2=2\cdot10^{11}\cdot4.52\cdot10^{-4}(0.352-0.05)^2=0.824\cdot10^7\,H\cdot m^2$$

$$D_{os,t}(\infty,50)=E_s'A_s'\left[h_0-x_0(\infty,50)\right]^2=2\cdot10^{11}\cdot32.17\cdot10^{-4}(0.6-0.352-0.05)^2=2.52\cdot10^7\,H\cdot m^2$$

$$D_0(\infty,50)=D_{ob,c}(\infty,50)+D_{ob,t}(\infty,50)+D_{os,c}(\infty,50)+D_{os,t}(\infty,50)=8.591\cdot10^7\,H\cdot m^2$$

According to Table 26 on page 77 of (Bondarenko and Suvorkin 1987), the value of f_0 is 0.13 for B35 concrete and f_0 is 0.12 for B30 concrete. The long-term deformation modulus of the upper edge section of the compression zone of the beam's cross-sectional area is determined using formula (15) from (Bondarenko and Suvorkin 1982) as follows.

$$E_b^f(\infty,50)=\left[\frac{1+v_k}{E_b(\infty)}+(1+v_c)C*(\infty,50)\right]^{-1}=$$

$$=\left[\frac{1+1.3}{3.569\cdot10^{10}}+(1+1{,}6)\cdot6.28\cdot10^{11}\right]^{-1}=0.439\cdot10^{10}\,H/m^2$$

Here, v_k=1.3 and v_c=1.6 are determined according to Table 9 from (Bondarenko and Suvorkin 1987). The effective deformation modulus of concrete in the compression zone of the reinforced concrete beam is determined using formula (14) from (Bondarenko and Suvorkin 1987).

$$E_b(\infty,50)=\frac{4+f_0}{(3+2f_0)E_b^f(\infty,50)}=\frac{4+0.13}{(3+2\cdot 0.13)\cdot 0.439\cdot 10^{10}}=0.556\cdot 10^{10}\,H/m^2$$

The limiting value of the relative height of the compression zone is determined according to formulas (23)–(24) from (Bondarenko and Suvorkin 1987).

$$\xi_R=\left[\frac{1+R_{sk}E_b^f(\infty,50)}{R_{bn}(28)E_s}\right]=\left[\frac{1+4\cdot 10^8\cdot 0.439\cdot 10^{10}}{22\cdot 10^6\cdot 2\cdot 10^{11}}\right]^{-1}=0.715;$$

$$\xi_R'=\frac{a'}{h_0}\left[1-\frac{R_{sk}'E_b^f(\infty,50)}{E_sR_{bn}(28)E_s}\right]=\frac{0.05}{0.55}\left[1-\frac{4\cdot 10^8\cdot 0.439\cdot 10^{10}}{22\cdot 10^6\cdot 2\cdot 10^{11}}\right]^{-1}=0.151,$$

here, R_{bn} = 22 MPa = 22 × 10^6 N/m² is taken from Table 3, page 8 of (Bondarenko and Suvorkin 1987). In the considered example

$$\frac{h_f'}{h_0}=\frac{0.06}{0.55}=0.109<\xi_R'=0.151$$

Since this is the case, the position of the neutral axis of the beam's cross-sectional area is determined using formula (25).

$$\frac{R_{bn}(28)B_f'h_f'}{1+f_0}+\frac{R_{bn}(28)}{E_b^f(\infty,50)}-\frac{h_f'-a}{h_f'E_s'A_s'}\begin{matrix}<\\>\end{matrix}R_{sk}A_s$$

$$\frac{22\cdot 10^6\cdot 0.35\cdot 0.06}{1+0.13}+\frac{22\cdot 10^6}{0.439\cdot 10^{10}}\cdot\frac{0.06-0.05}{0.06\cdot 2\cdot 10^{11}\cdot 4.52\cdot 10^{-4}}=4.844\cdot 10^5\,H<R_{sk}A_s$$

$$=4\cdot 10^8\cdot 32.17\cdot 10^{-4}=12.87\cdot 10^5\ H$$

Since $\xi_R'=0.151<\xi_R=0.715$, the height of the compression zone of the beam's cross-sectional area is determined based on (Bondarenko and Suvorkin 1987) formula (27) using the Newton-Raphson method.

$$F[x_{min}(\infty,50)]=ax_{min}(\infty,50)+bx_{min}(\infty,50)\left[1-\frac{1-h_f'}{x_{min}(\infty,50)}\right]^{1+f_0}-c=0,$$

here $\sigma_s=R_{sk}$, $\sigma_s'=R_{sk}'$.

$$a=\frac{R_{bn}(28)b}{1+f_0}=\frac{22\cdot 10^6\cdot 0.2}{1+0.13}=3.893\cdot 10^6;$$

$$b=\frac{R_{bn}(28)(b_f'-b)}{1+f_0}=\frac{22\cdot 10^6\cdot(0.35-0.2)}{1+0.13}=2.92\cdot 10^6;$$

$$c=R_{sk}-R_{sk}'A_s'=4\cdot 10^8\cdot 32.17\cdot 10^{-4}-4\cdot 10^8\cdot 4.52\cdot 10^{-4}=1.106\cdot 10^6.$$

The first term on the left side of this equation, with respect to $x_{min}(\infty,50)$, yields the following equation.

$$F'[x_{min}(\infty,50)]=a+b\left[1-\left(\frac{h_f'}{x_{min}(\infty,50)}\right)^{1+f_0}\right]-b(1+f_0)\left(1-\frac{h_f'}{x_{min}(\infty,50)}\right)\frac{h_f'}{x_{min}(\infty,50)}.$$

Now, when $x_{min}^0(\infty,50)=0.5h=0.5\cdot 0.6=0.3$.

$$F[x_{min}^0(\infty,50)]=3.893\cdot 10^6\cdot 0.3+2.92\cdot 10^6\cdot 0.3\cdot\left(1-\frac{1-0.06}{0.3}\right)^{1+0.13}-1.106\cdot 10^6=0.257\cdot 10^6;$$

$$F'\left[x_{min}^{0}(\infty,50)\right]=3.893\cdot10^{6}+2.92\cdot10^{6}\cdot\left[1-\left(1-\frac{1-0.06}{0.3}\right)^{1+0.13}\right]-$$

$$-2.92\cdot10^{6}\cdot\left(1-\frac{0.06}{0.3}\right)^{1+0.13}\cdot\frac{0.06}{0.3}=4.031\cdot10^{6}$$

According to the first approximation, the following value is determined.

$$x'_{min}(\infty,50)=x_{min}^{0}(\infty,50)-\frac{F\left[x'_{min}(\infty,50)\right]}{F'\left[x_{min}^{0}(\infty,50)\right]}=0.3-\frac{0.257\cdot10^{6}}{4.031\cdot10^{6}}=0.236\,m$$

Assuming $x'_{min}(\infty,50)=0.236$ m, the following result is obtained in the second approximation.

$$F(x'_{min})=3.893\cdot10^{6}\cdot0.236+2.92\cdot10^{6}\cdot0.236\cdot\left[1-\left(\frac{1-0.06}{0.236}\right)^{1+0.13}\right]-1.106\cdot10^{6}=7.18\cdot10^{3}$$

$$F'(x'_{min})=3.893\cdot10^{6}+2.92\cdot10^{6}\cdot\left[1-\left(1-\frac{0.06}{0.236}\right)^{1+0.13}\right]-$$

$$-2.92\cdot10^{6}\cdot(1+0.13)\left(1-\frac{0.06}{0.236}\right)^{1+0.13}\cdot\frac{0.06}{0.3}=4.293\cdot10^{6}$$

$$x_{min}^{2}(\infty,50)=x'_{min}(\infty,50)-\frac{E(x'_{min})}{F'(x'_{min})}=0.236-\frac{7.18\cdot10^{3}}{4.293\cdot10^{6}}=0.234\,m$$

Considering the insignificant difference between the values of x_{min}^{1} and x_{min}^{2}, we accept $x_{min}(\infty, 50) = 0.234$ m. Now, we check the following condition.

$$\xi'_R=0.15<\xi=\frac{x_{min}(\infty,50)}{h_0}=\frac{0.234}{0.55}=0.425\,m<\xi_R=0.715$$

Thus, based on formula (Bondarenko and Suvorkin 1987) (31), we fully accept the value of $x_{min}(\infty, 50)$. Now, taking into account formulas (Bondarenko and Suvorkin 1987) (16) and (17), we determine the coefficients of the non-linear deformation of the reinforcement, β_s and β'_s. From Table 3 in (Bondarenko and Suvorkin 1987), we select the ultimate compressive strain value for B30 grade concrete as $\varepsilon_{bn}=1.38\cdot10^{-3}$. The values of $\varepsilon_s(\infty, 50)$ and $\varepsilon'_s(\infty,50)$ are determined as follows.

$$\varepsilon_s(\infty,50)=1.864\cdot10^{-3}.\ \varepsilon'_s(\infty,50)=\frac{\varepsilon_{bn}\left(x_{min}(\infty,50)-a'\right)}{x_{min}(\infty,50)}=1.085\cdot10^{-3}.$$

According to formulas (Bondarenko and Suvorkin 1987) (16) and (17), the following coefficients are determined.

$$\beta_s=\frac{R_{sk}}{E_s\varepsilon_s(\infty,50)}=\frac{4\cdot10^{8}}{2\cdot10^{11}\cdot1.864\cdot10^{-3}}=1.073;$$

$$\beta'_s=\frac{R_{sk}}{E'_s\varepsilon'_s(\infty,50)}=\frac{4\cdot10^{8}}{2\cdot10^{11}\cdot1.085\cdot10^{-3}}=1.843$$

Thus, since $\beta_s=1.073$ and $\beta'_s=1.843$ are greater than 1, in subsequent calculations these values are assumed to be equal to 1, and based on formula (Bondarenko and Suvorkin 1987) (13), $q_o(\infty, 50)$ is determined.

$$q_0(\infty,50)=\frac{1}{2}\frac{E_b(\infty,50)\left[bx_{min}^{2}(\infty,50)+(b'_f-b)h_f^{1}+\beta_sE_sA_sh_0+\beta'_sE'_sA'_sa'\right]}{E_b(\infty,50)\left[bx_{min}(\infty,50)+(b'_f-b)h_f^{1}+\beta_sE_sA_s+\beta'_sE'_sA'_s\right]}=0.37\,m=37\,cm$$

Transverse section rigidity based on the limiting stress condition according to formulas (9)–(12).

$$D_{min\,b,c}(\infty,50)=E_b(\infty,50)\left[\frac{bx_{min}^3(\infty,50)}{12}+bx_{min}(\infty,50)\left(q(\infty,50)-\frac{x_{min}(\infty,50)}{2}\right)^2+\right.$$

$$\left.+\frac{(b'_f-b)h_f^{13}}{12}+(b'_f-b)h'_f\left(q_0(\infty,50)-\frac{h_f}{2}\right)^2\right]=0.556\cdot10^{10}\left[\frac{0.2\cdot0.234^3}{12}+\right.$$

$$\left.+0.2\cdot0.234\left(0.37-\frac{0.234}{2}\right)^2+(0.35-0.2)\frac{0.06^3}{12}+(0.35-0.2)\cdot0.06\left(0.37-\frac{0.06}{2}\right)^2\right]=$$

$$=2.36\times10^7\ H{\cdot}m^2;$$

$$D_{s,c}(\infty,50)=\beta'_sE'_sA'_s(q_0(\infty,50)-a')^2=0.93\times10^7\ H\cdot m^2;$$

$$D_{s,t}(\infty,50)=\beta_sE_sA_s(h_0-q_0(\infty,50))^2=2.08\times10^7\ H\cdot m^2;$$

$$D_{min}(\infty,50)=D_{min\,b,c}(\infty,50)+D_{s,c}(\infty,50)+D_{s,t}(\infty,50)=5.37\times10^7\ H\cdot m^2.$$

We determine the limiting value of the bending moment according to formula (33) from (Bondarenko and Suvorkin 1987).

$$M_{max}(\infty,50)=\frac{R_{bn}(28)x_{min}^2(\infty,50)}{f_0\left[b+(b'_f-b)\left(1+\frac{1-h'_f}{x_{min}(\infty,50)}\right)^{2+f_0}\right]}+R_{sk}A_s(h_0-x_{min}(\infty,50))$$

$$+R'_{sk}A'_s(x_{min}(\infty,50)-a')=\frac{22\cdot10^6\cdot0.234^2}{(2+0.13)\left(0.2+(0.35-0.2)\left[1-\left(\frac{0.06}{0.234}\right)^{2+0.13}\right]\right)}+$$

$$+400\cdot10^6\cdot32.17\cdot10^{-4}\cdot(0.55-0.234)+4.52\cdot10^{-4}\cdot400\cdot10^6(0.234-0.05)=$$

$$=5.93\times10^5\ H\cdot m^2$$

The rigidity of the mid-span section of the beam arch is determined as follows according to formula (1).

$$D(\infty,50)=k(\infty-50)D_0(\infty,50)\left(1-\frac{1-D_{min}(\infty,50)}{k(\infty-50)D_0(\infty,50)}\cdot\frac{M}{M_{max}(\infty,50)}\right),$$

here $k(\infty-50)=1,\ \ t-t_0=\infty-50>60\,\text{day}$

$$D(\infty,50)=1\cdot8.591\cdot10^7\left(1-\frac{1-5.37\cdot10^7}{1\cdot8.591\cdot10^7}\cdot\frac{31.5\cdot10^4}{59.3\cdot10^4}\right)=6.88\times10^7\ H\cdot m^2$$

We determine the bending value at the mid-span of the beam arch according to formula (35) from (Bondarenko and Suvorkin 1987).

$$L(\infty,50)=\frac{2D_0(\infty,50)k(\infty-50)M_{max}(\infty,50)}{q(k_0(\infty,50)D_0(\infty,50)-D_{min}(\infty,50))}=\frac{2\cdot1\cdot8.591\cdot10^7\cdot5.93\cdot10^5}{70\cdot10^3\cdot(1\cdot8.591\cdot10^7-5.37\cdot10^7)}=45{,}19\,m^2$$

$$f_m(\infty)=\frac{M_{max}(\infty,50)}{k_0(\infty-50)D_0(\infty,50)-D_{min}(\infty,50)}<-\frac{L^2}{8}+\frac{L(\infty,50)}{2}\left[ln\left(1-\frac{L^2}{4L(\infty,50)}\right)+\right.$$

$$\left.+\frac{2L}{\sqrt{4L(\infty,50)-L^2}}\frac{arctgL}{4L(\infty,50)-L^2}\right]>=\frac{5.93\cdot 10^5}{1\cdot 8.591\cdot 10^7\cdot 5.37\cdot 10^7}<\frac{6^2}{8}+$$

$$+\frac{45{,}19}{2}\left[ln\left(1-\frac{6^2}{4\cdot 45.19}\right)+\frac{2\cdot 6}{\sqrt{4\cdot 45.19-6^2}}\frac{arctg6}{\sqrt{4\cdot 45.19-6^2}}\right]>=0.182\ m$$

Conclusion

The practical solution presented in the article is based on the computational models recommended by regulatory norms. The rigidity and bending of reinforced concrete beams with a cross-sectional T-shape were determined, taking into account the slip properties of concrete. Based on this calculation sequence, experimental and theoretical research problems can be analyzed, leading to scientific conclusions.

References

1. ACI Committee 318. (2019). Building Code Requirements for Structural Concrete (ACI 318-19).
2. Atmajayanti, A.T., et al. (2025). Effective Flexural Strengthening of Reinforced Concrete T-Beams Using Bonded Fiber-Core Steel Wire Ropes. Fibers, 13(5), 53.
3. Bazant, Z.P. (2001). *Prediction of concrete creep and shrinkage: past, present and future*. Nuclear Engineering and Design. 203, 27–38
4. Bazant, Z.P. and Baweja, S. (1995). Creep and shrinkage prediction model for concrete structures.
5. Bondarenko V.M. and Bondarenko S.V. (1982). Engineering Methods of Nonlinear Theory of Reinforced Concrete. Moscow: Vysshaya Shkola.
6. Bondarenko V.M. and D.G. Suvorkin. (1987). Reinforced Concrete and Calonet Structures. Moscow: Vysshaya Shkola.
7. CEB-FIP Model Code 2010. Design Code for Concrete Structures.
8. Elaghoury, Z. & Bartlett, F.M. (2019). Long-Term Deflections of Reinforced Concrete Beams. CSCE Annual Conference. June 12 - 15,
9. Eurocode 2. (2004). Design of Concrete Structures.
10. Gardner, N.J. (2004). Comparison of creep and shrinkage prediction models.
11. Gardner, N.J., & Lockman, M.J. (2001). Design provisions for drying shrinkage and creep of concrete. ACI Materials Journal, 98(2), 123–132.
12. Gilbert, R.I. (1988). Time-dependent behaviour of reinforced concrete slabs.
13. Jeong, H., et al. (2022). Development of Mapping Function to Estimate Bond–Slip and Bond Strength of RC Beams. International Journal of Concrete Structures and Materials. 16(49). https://doi.org/10.1186/s40069-022-00536-6
14. Kosimov T. (2019). Stress-strain state of iron-concrete creep. Monograph. Samarkand.
15. Lin, Y. (2010). Tension Stiffening Model for Reinforced Concrete Based on Bond Stress-Slip Relation. Penn State University.
16. Neville, A.M. (1997). Properties of Concrete.
17. Prokopovich I.E. and Zadgenidze V.A. (1980). Applied Creep Theory. Moscow: Stroyizdat.
18. Rahaf Mohamad, G., Wardeh, M., Al Ahmad Al Kousa, A., Jahami. (2025). 1D Finite Element Modeling of Bond-Slip Behavior. Designs, 9(3), 75. DOI: 10.3390/designs9030075.
19. Recommendation on creep and shrinkage (1988). Moscow Guidelines.
20. RILEM TC 107. (1999). Shrinkage of Concrete.
21. SP63.13330.2018. Concrete and Reinforced Concrete Structures.

Figure 1. Calculation diagram of the beam (a) and the cross-sectional geometry (b).

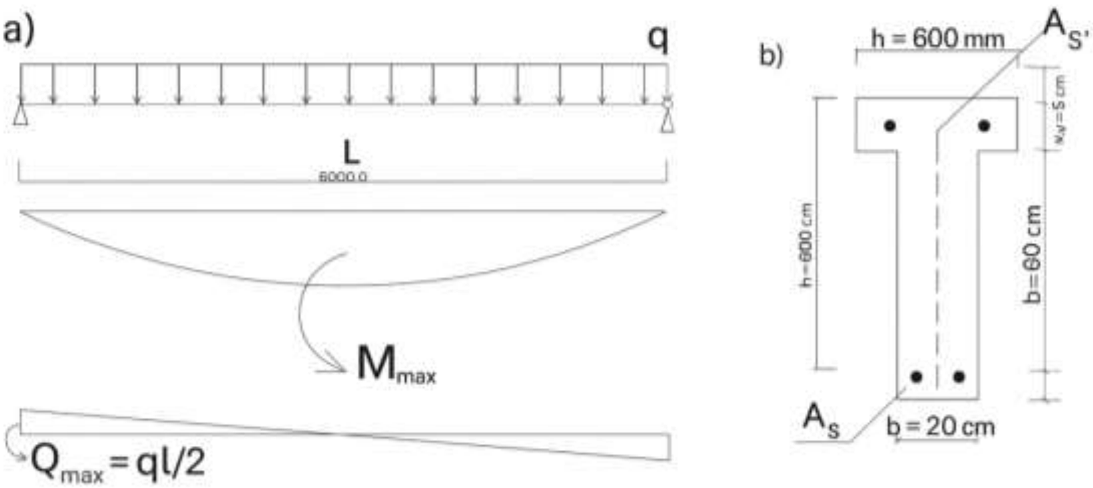

Table 1

Deflection and Stiffness of RC T-Beam Over Time

Time (day)	Deflection (mm)	Stiffness (kNm2)
50	5	23000
100	8	22000
200	12	21000
400	18	20000
800	25	19000
1600	32	18000
Infinity	40	17000

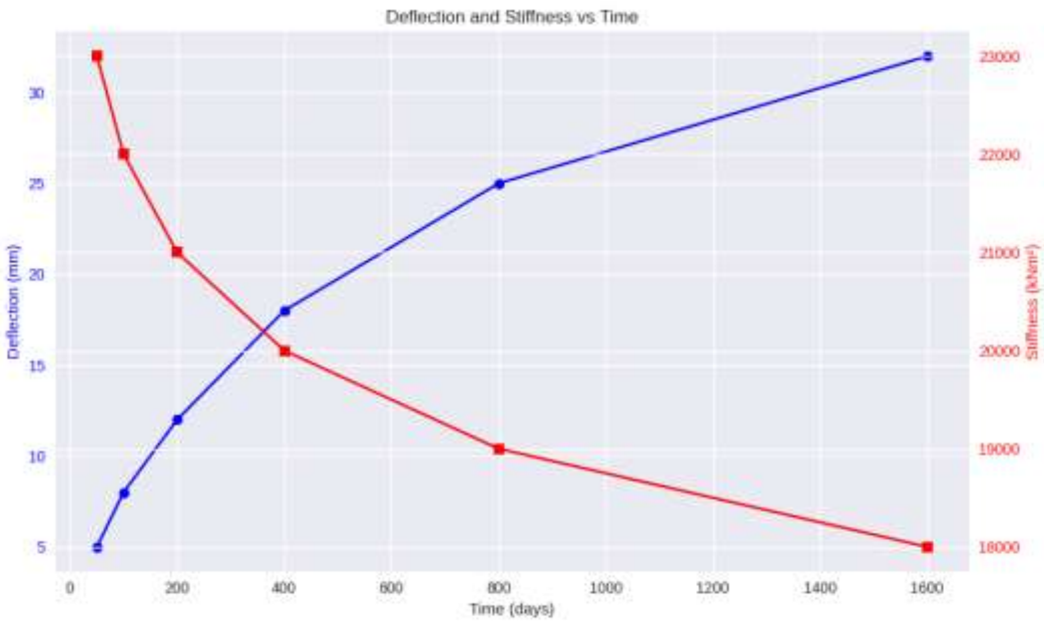

Table 2

Comparison of Deflection and Stiffness: Perfect Bond vs Bond-Slip

Time (days)	Deflection (mm)-Perfect Bond	Deflection (mm)-Bond-Slip	Stiffness (kNm2)-Perfect Bond	Stiffness (kNm2)-Bond-Slip
50	4	5	24000	23000
100	6	8	23000	22000
200	9	12	22000	21000
400	14	18	21000	20000
800	20	25	20000	19000
1600	26	32	19000	18000
Infinity	34	40	18000	17000